

Decoherence induced by long wavelength gravitons

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Abstract

We discuss how a background bath of gravitons can induce decoherence of quantum systems. The mechanism is dephasing, the loss of phase coherence due to quantum geometry fluctuations caused by the gravitons. This effect is illustrated in a simple analog model of quantum particles in a cavity whose walls undergo position fluctuations, and create the same effect expected from spacetime geometry fluctuations. We obtain an explicit result for the decoherence rate in the limit where the graviton wavelength is large compared to the size of the quantum system, and make some estimates for this rate.

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I. INTRODUCTION

The interaction of the environment with a quantum system tends to lead to a loss of quantum coherence (decoherence), and a transition from quantum to classical behavior. Decoherence has been the topic of extensive investigation in recent years. (For a review, see Ref. [1].) Several authors [2–4] have suggested that the gravitational interaction might play a special role in the quantum to classical transition, but this view remains controversial. For a recent review with further references, see Hu [5]. However, there seems to be no doubt that gravitational interactions can contribute to decoherence.

In this paper, we will treat systems where the role of gravity is to provide spacetime geometry fluctuations which in turn lead to length and relative phase fluctuations. This effect can arise from the presence of a bath of long wavelength gravitons. First consider gravity waves on a flat background described by a metric of the form (in $c = 1$ units)

$$ds^2 = -dt^2 + h_{ij} dx^i dx^j, \quad (1)$$

where the transverse tracefree gauge is assumed. Fluctuations of the metric lead to fluctuations of the squared proper length $d\ell^2 = h_{ij} dx^i dx^j$, and hence of the separation between nearby geodesics. This follows from the geodesic deviation equation for the separation ξ^i ,

$$\frac{d^2 \xi_i}{dt^2} = -R_{itjt} \xi^j, \quad (2)$$

where the relevant component of the Riemann tensor is given by $R_{itjt} = -\frac{1}{2}\partial_t^2 h_{ij}$ in the transverse tracefree gauge. In many situations, length fluctuations can lead to quantum phase fluctuations, and hence decoherence through dephasing. In Sec. II, we will treat a simple model in which the length fluctuations arise from fluctuating boundaries, which forms an analog model for the effects of quantum geometry fluctuations. We apply the lessons of this model to the effects of a graviton bath in Sec. III, and obtain estimates for the decoherence rate. Our results are summarized in Sec. IV.

II. A MODEL WITH FLUCTUATING BOUNDARIES

We begin with a non-relativistic particle of mass m confined in an infinite potential well whose width is denoted by a . For the sake of simplicity, we restrict our analysis to the case of one space dimension. Suppose that the normalized state of the particle at an arbitrary time t is given by a superposition of the first two available states as

$$\psi(x, t) = \frac{1}{\sqrt{2}} [\psi_1(x, t) + \psi_2(x, t)]. \quad (3)$$

The eigenfunctions ψ_n ($n = 1, 2$) are independent solutions of the Schrödinger equation under the conditions that these functions vanish on the boundaries at $x = 0$ and $x = a$, yielding

$$\psi_n(x, t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) e^{-i\omega_n t}, \quad (4)$$

with $\omega_n = n^2\pi^2\hbar/2ma^2$. Each one of these eigenfunctions describes a stationary state, as the corresponding probability density $|\psi_n|^2$ is time-independent. However, time evolution

occurs when the particle is governed by a linear superposition of $\psi_n(x, t)$, such as that given in Eq. (3). The probability density can be obtained as

$$|\psi(x, t)|^2 = \frac{1}{2} (|\psi_1|^2 + |\psi_2|^2) + |\psi_1||\psi_2| \cos(\Delta\omega t), \quad (5)$$

where we define $\Delta\omega = \omega_2 - \omega_1$, the Bohr angular frequency associated with the energy difference of the superposed states. As we see, the time evolution of $|\psi(x, t)|^2$ is exclusively governed by the interference term between ψ_1 and ψ_2 . In fact, $|\psi(x, t)|^2$ oscillates between its maximum $(|\psi_1| + |\psi_2|)^2/2$ and minimum $(|\psi_1| - |\psi_2|)^2/2$ values, as depicted in the down inset frame in Fig. 1, for a particular numerical model.

Now we wish to investigate the behavior of this quantum system when interaction with the environment takes place. In order to model the interaction between the system and its environment, we allow the positions of the physical boundaries to fluctuate under the influence of an external noise. This can simply be implemented by allowing the width parameter a to undergo fluctuations around a mean value \bar{a} [6]. Note that fluctuations in the width, a , lead to fluctuations in the energy levels, ω_n .

An accelerating boundary can emit quantum radiation which might interact with the particle [9–11]. However, quantum fluctuations of position need not imply classical acceleration or radiation. We view the boundary as being analogous to an electron in a quantum state, such as a Gaussian wavepacket, which is not an eigenstate of position, but yet need not radiate.

We set $a = \bar{a}(1 + \varepsilon)$, where the dimensionless parameter ε is described by a Gaussian distribution as

$$f(\varepsilon) = \sqrt{\frac{\theta}{\pi}} e^{-\theta\varepsilon^2}, \quad (6)$$

with θ related to the width σ of the distribution by means of $\sigma^2 = 1/(2\theta)$. The mean value of an arbitrary function $G(\varepsilon)$ over ε is a linear operation defined by

$$\langle G \rangle = \int_{-\infty}^{\infty} G(\varepsilon) f(\varepsilon) d\varepsilon. \quad (7)$$

Notice that $\langle \varepsilon^2 \rangle - \langle \varepsilon \rangle^2 = \sigma^2$, the mean squared fluctuation of ε .

There does not seem to be a meaning to averaging the wave function $\psi(x, t)$, as it is not directly observable. (See, however, Ref. [12] for a discussion of the possibility of measuring a wave function in the context of a weak measurements approach.) Here we are interested in studying averaged values of observable quantities. Particularly, the modulus squared of the particle wave function, Eq. (5), represents the probability density associated with the position the particle. The average over width fluctuations of this quantity can be calculated by using Eqs. (5) and (7), yielding

$$\langle |\psi|^2 \rangle = \frac{1}{2} (\langle |\psi_1|^2 \rangle + \langle |\psi_2|^2 \rangle) + \langle |\psi_1||\psi_2| \cos(\Delta\omega t) \rangle. \quad (8)$$

Here the angular bracket refers to the average over positions of the boundaries, which was defined in Eq. (7). In what follows, each term appearing in the above equation will be considered separately.

The two first terms appearing in Eq. (8) can be expressed, using Eqs. (4) and (7), as

$$\langle |\psi_n|^2 \rangle = \sqrt{\frac{4\theta}{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\theta\varepsilon^2}}{\bar{a}(1 + \varepsilon)} \sin^2 \left[\frac{n\pi x}{\bar{a}(1 + \varepsilon)} \right] d\varepsilon, \quad (9)$$

where $n = 1, 2$. We assume a narrow distribution, $\sigma \ll 1$, which means that only small values of ε contribute in Eq. (9). We expand the integrand, apart from the exponential, to second order in ε , and then integrate to obtain

$$\begin{aligned} \langle |\psi_n|^2 \rangle = & \frac{2}{\bar{a}} \sin^2 \left(\frac{n\pi x}{\bar{a}} \right) + \left[\frac{2}{\bar{a}} \sin^2 \left(\frac{n\pi x}{\bar{a}} \right) + \frac{4n\pi x}{\bar{a}^2} \sin \left(\frac{2n\pi x}{\bar{a}} \right) \right. \\ & \left. + \frac{2n^2\pi^2 x^2}{\bar{a}^3} \cos \left(\frac{2n\pi x}{\bar{a}} \right) \right] \sigma^2 + O(\sigma^4). \end{aligned} \quad (10)$$

Further, as the particle is confined ($0 \leq x \leq \bar{a}$), and the small σ approximation, $\pi\sigma x/\bar{a} \ll 1$, is assumed, we have

$$\langle |\psi_n|^2 \rangle \approx \frac{2}{\bar{a}} \sin^2 \left(\frac{n\pi x}{\bar{a}} \right) + O(\sigma^2) \approx |\psi_n|^2. \quad (11)$$

In lowest order in σ , the probability density associated with the energy eigenstates does not change when boundary fluctuations are introduced. As may be seen from Eq. (10), there is a shift in this density in order σ^2 . However, this shift is time-independent, and not of special interest here. If the particle is initially in an energy eigenstate state $\psi_n(x, t)$, it will remain in this state and its probability density will not undergo appreciable time evolution when small boundary fluctuations are present.

Next, we consider the last term in Eq. (8), which describes the interference effects occurring in the system. From Eqs. (4) and (7), we obtain that

$$\begin{aligned} \langle |\psi_1||\psi_2| \cos \omega t \rangle = & \frac{1}{4\bar{a}} \sqrt{\frac{\theta}{\pi}} (A_1^+ + A_{-1}^+ - A_3^+ - A_{-3}^+ \\ & + A_1^- + A_{-1}^- - A_3^- - A_{-3}^-), \end{aligned} \quad (12)$$

where A_q^\pm is defined by

$$A_q^\pm = \int_{-\infty}^{\infty} \frac{1}{1+\varepsilon} \exp \left[\frac{qi\pi x}{\bar{a}(1+\varepsilon)} \pm \frac{i\Delta\omega t}{(1+\varepsilon)^2} - \theta\varepsilon^2 \right] d\varepsilon, \quad (13)$$

and $\Delta\omega = \omega_2(\bar{a}) - \omega_1(\bar{a}) = 3\pi^2/2m\bar{a}^2$ is the energy difference of the two levels. Once the small σ approximation is assumed, in order to solve the above integral it is enough to Taylor expand to first order in ε inside the exponential, but only to zeroth order otherwise. Thus, using the identity

$$\int_{-\infty}^{\infty} e^{\pm iZ\varepsilon - \theta\varepsilon^2} d\varepsilon = \sqrt{\frac{\pi}{\theta}} e^{-Z^2/4\theta}. \quad (14)$$

and neglecting terms which become unimportant after a finite time $t \gtrsim 1/\Delta\omega$, it follows that

$$A_q^\pm \approx \sqrt{\frac{\pi}{\theta}} \exp \left[\frac{iq\pi}{\bar{a}} \left(x \pm \frac{\bar{a}\Delta\omega t}{q\pi} \right) \right] e^{-\Gamma t^2}, \quad (15)$$

where $\Gamma = 2\Delta\omega^2\sigma^2$. Using this result in Eq. (12), we obtain

$$\langle |\psi_1||\psi_2| \cos \omega t \rangle = \frac{4}{\bar{a}} \cos \left(\frac{\pi x}{\bar{a}} \right) \sin^2 \left(\frac{\pi x}{\bar{a}} \right) \cos(\Delta\omega t) e^{-\Gamma t^2}. \quad (16)$$

As one can see this term describes oscillations modulated by a factor which decays exponentially in squared time. The time scale for the onset of this decay is

$$t_o = \frac{1}{\Delta\omega} . \quad (17)$$

In the case of an electron in a potential well with $\bar{a} \approx 1\text{\AA}$, this time is of order $t_o \approx 10^{-17} \text{ s}$. However, once the decay begins, the characteristic decay time is

$$t_d = \frac{1}{\sqrt{\Gamma}} = \frac{t_o}{\sqrt{2}\sigma} , \quad (18)$$

which is longer by a factor of about $1/\sigma$.

Combining the results in Eqs. (11) and (16) with Eq. (8), we find that the average $\langle |\psi(x, t)|^2 \rangle$ becomes

$$\langle |\psi|^2 \rangle \approx \frac{1}{2} (|\psi_1|^2 + |\psi_2|^2) + \frac{4}{\bar{a}} \cos\left(\frac{\pi x}{\bar{a}}\right) \sin^2\left(\frac{\pi x}{\bar{a}}\right) \cos(\Delta\omega t) e^{-\Gamma t^2} . \quad (19)$$

As time passes, the last term in the above equation falls to zero for $t \gg t_d$. Thus, the net effect of the fluctuations is to kill the interference term. Neglecting the last term in Eq. (19) we obtain that

$$\langle |\psi|^2 \rangle = \frac{1}{2} (|\psi_1|^2 + |\psi_2|^2) , \quad (20)$$

which corresponds to a weighted sum of probabilities as occurs when a statistical mixture of states is considered.

The behavior described by Eq. (19) can be illustrated by a numerical example. For instance, let us study the time evolution of the averaged probability density when fluctuating boundaries are considered in a particular model with $x/\bar{a} = 0.7$ and $\sigma = 0.01$. In this case Eq. (8) can be integrated numerically. The result is depicted in Fig. 1. Alternatively we could have used the approximate solution given by Eq. (19), which leads to an identical graph, confirming the approximation used in obtaining Eq. (19). As the figure shows clearly, the probability density $\langle |\psi|^2 \rangle$ oscillates around the mean value $(\langle |\psi_1|^2 \rangle + \langle |\psi_2|^2 \rangle)/2$, converging to this value when $t \gg t_d$. As anticipated, the net effect of the fluctuating boundaries of the potential well is to kill the interference effect between the two state components ψ_1 and ψ_2 . Finally, when no fluctuations are present, the usual stationary solution holds, as illustrated by the down inset frame in Fig. 1.

In order to have an estimate, consider again the case of an electron in a potential well with $\bar{a} \sim 1\text{\AA}$. As shown in Fig. 1 the oscillations in $\langle |\psi|^2 \rangle$ are completely suppressed when $\bar{\omega}t = 200$, that is, 10^{-14} seconds after the boundary fluctuations are turned on.

III. SPACETIME GEOMETRY FLUCTUATIONS FROM A BATH OF GRAVITONS

Now we wish to turn to quantum systems where the length fluctuations are due to space-time geometry fluctuations. However, we first review the effects of a classical gravity wave, described by the metric in Eq. (1), which can cause variations in the positions of a boundary.

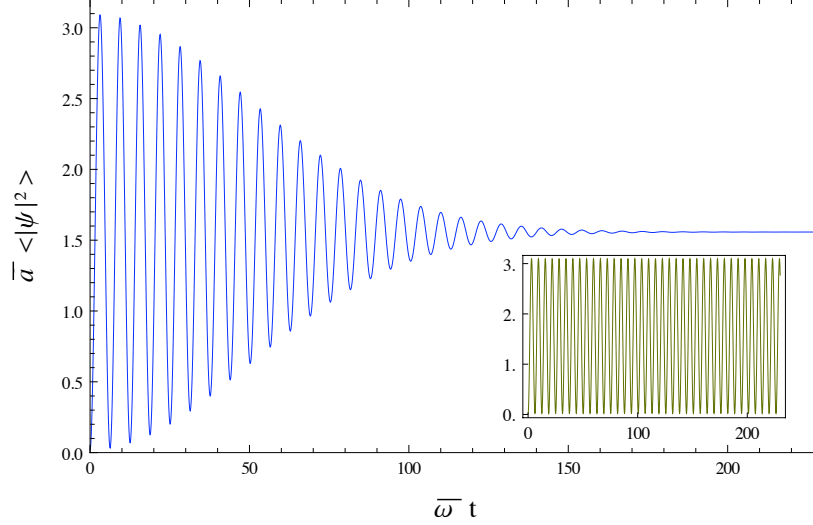


FIG. 1: (color online). The figure shows the behavior of the probability density associated with the state defined by Eq. (3) when fluctuating boundaries are assumed. As time goes on, interference effects between the component states of $\psi(x, t)$ are suppressed. For fixed boundaries, no suppression of interference is found, as shown in the down inset frame. We set $x/\bar{a} = 0.7$ and $\sigma = 0.01$.

The right hand side of Eq. (2) is the tidal acceleration, or force per unit mass on the walls of the cavity due to gravity. Non-gravitational forces will modify this equation with the addition of other terms. Consider the case where the walls are bound in a harmonic potential with natural frequency ω_0 . If the walls are displaced from their equilibrium position by a distance $\delta\xi$, then the restoring force per unit mass is $-\omega_0^2 \delta\xi$. In the transverse, tracefree gauge, the Riemann tensor for the spacetime described by Eq. (1) may be expressed as

$$R_{itjt} = -\frac{1}{2} \ddot{h}_{ij}. \quad (21)$$

Let the displacements be in the x -direction, and write $\xi^i = \delta_x^i \xi$. Now Eq. (2) is modified to

$$\ddot{\xi} = \delta\ddot{\xi} = \frac{1}{2} \ddot{h}_{xx} \xi - \omega_0^2 \delta\xi. \quad (22)$$

This is the equation for a driven harmonic oscillator, with damping effects neglected. Let ω be the frequency of the gravity wave and hence also of the response $\delta\xi$, so $\ddot{h}_{xx} = -\omega^2 h_{xx}$, and $\delta\ddot{\xi} = -\omega^2 \delta\xi$. This leads to a result for the fractional change in position of the wall:

$$\frac{\delta\xi}{\xi} = \frac{1}{2} h_{xx} \left(\frac{\omega^2}{\omega^2 - \omega_0^2} \right). \quad (23)$$

Because damping effects have been ignored, this result is not expected to hold near resonance, $\omega \approx \omega_0$, but can be a good approximation well away from resonance. In particular, if the gravity wave frequency is above the natural frequency of the bound system, $\omega \gg \omega_0$, then the magnitude of the fractional change in position is of the same order as the metric perturbation

$$\left| \frac{\delta\xi}{\xi} \right| \approx \frac{1}{2} h_{xx}. \quad (24)$$

Note that this includes walls moving on geodesics as the special case where $\omega_0 = 0$. If the gravity wave frequency is below resonance, $\omega \ll \omega_0$, then $|\delta\xi/\xi|$ is suppressed by a factor of $(\omega/\omega_0)^2$, so we will focus on the former case described by Eq. (24).

The use of the geodesic deviation equation, Eq. (2), to describe the relative motion of the components of a quantum system, such as the boundaries of a cavity, assumes that the wavelength of the gravity wave is larger than the geometric size of the system. This arises because we are assuming that the Riemann tensor is approximately constant on a length scale ξ in writing Eq. (2). Thus the gravity wave frequency is bounded both from above and from below:

$$\frac{2\pi}{\xi} > \omega > \omega_0. \quad (25)$$

Now we wish to replace the classical gravity wave with a fluctuating spacetime geometry. One way to do this is with a bath of gravitons. We will use some results obtained in Refs. [13–15]. Suppose that gravitons are in a state where

$$\langle h_{ij} \rangle = 0 \quad (26)$$

but

$$h^2 = \frac{1}{9} \langle h_{ij} h^{ij} \rangle \neq 0. \quad (27)$$

Examples of such a state include thermal states and squeezed vacuum states. If the characteristic frequency of the gravitons satisfies Eq. (25), the root-mean-square fractional length fluctuations are of order h , which now plays the role of the parameter σ in the previous section. The factor of $1/9$ in Eq. (27) is motivated by the expectation that in an isotropic bath with all polarization states equally excited, we will have $\langle h_{xx}^2 \rangle = \langle h_{yy}^2 \rangle = \langle h_{zz}^2 \rangle = \langle h_{xy}^2 \rangle = \langle h_{yx}^2 \rangle = \langle h_{xz}^2 \rangle = \langle h_{zx}^2 \rangle = \langle h_{yz}^2 \rangle = \langle h_{zy}^2 \rangle$, and hence $h^2 = \langle h_{xx}^2 \rangle$. Then h will be the root-mean-square fractional length fluctuations in a given direction, such as the x -direction. [Note that h^2 was defined with a different numerical factor in Refs. [14, 15].]

In a graviton bath, there will be quantum phase fluctuations, just as in the model in Sect II, leading to dephasing and a loss of contrast in an interference pattern. If the probability distribution for the quantum metric fluctuations is approximately Gaussian, then the contrast will decay as an exponential of the squared time, as in Eq. (16). This leads to essentially the same decoherence time as in the fluctuating boundary model

$$t_d \approx \frac{1}{h \Delta\omega}. \quad (28)$$

As before, $\Delta\omega$ is the energy difference between the interfering states. Even if the probability distribution is not Gaussian, Eq. (28) is a reasonable estimate for the decoherence time, although the decay rate may not have the functional form of Eq. (16).

Consider the example of a thermal state of gravitons at temperature T . In Ref. [14], it was shown that for such a state

$$\langle h_{ij} h^{ij} \rangle = \frac{16}{3} \pi \ell_P^2 T^2, \quad (29)$$

where ℓ_P is the Planck length. (Note that units in which $32\pi G = 32\pi\ell_P^2 = 1$, where G is Newton's constant, were used in Ref. [14].) This leads to an estimate of the decoherence rate given by

$$\frac{1}{t_d} \approx \frac{4}{3} \sqrt{\frac{\pi}{3}} T \ell_P \Delta\omega = \frac{8\pi}{3} \sqrt{\frac{\pi}{3}} T \frac{\Delta\omega}{E_P}. \quad (30)$$

Here $E_P = 2\pi/\ell_P$ is the Planck energy, and we are using units with $\hbar = k_B = 1$, where k_B is Boltzmann's constant.

This result may be compared with a formula recently given by Blencowe [16] for the decoherence rate by a thermal bath of gravitons

$$\left(\frac{1}{t_d}\right)_{\text{Blencowe}} = T \left(\frac{\Delta\omega}{E_P}\right)^2. \quad (31)$$

This apparent discrepancy probably arises from use of different assumptions. Our result, Eq. (30), assumes a low temperature limit in the sense that the graviton wavelengths must be long compared to the geometric dimensions of the quantum system. In contrast, Blencowe assumes a high temperature limit. In a regime where our result, Eq. (30) is applicable, it predicts a larger decoherence rate than does Eq. (31). This arises because gravitons with wavelengths short compared to the size of the system can be expected to be less effective in producing dephasing. We may write the decoherence time predicted by Eq. (30) as

$$t_d \approx 2 \times 10^9 \text{yr} \left(\frac{1\text{K}}{T}\right) \left(\frac{1\text{eV}}{\Delta\omega}\right). \quad (32)$$

Thus the gravitational decoherence rate is very small unless either the energy difference of the interfering states, or the effective graviton temperature is large.

A truly thermal bath of gravitons could be difficult to produce due to the weak coupling of gravitons to one another and to matter, leading to very long equilibration times. Hawking radiation from black holes [17] is one mechanism to produce thermal gravitons. In addition, quantum particle creation in an expanding universe can sometimes produce an approximately Planckian spectrum of particles [18], including gravitons. Quantum creation of gravitons and other particles are expected at the end of an inflationary era, and could contribute significantly to the matter and radiation in the universe after the end of inflation [19]. In addition, quantum stress tensor fluctuations during inflation might contribute to the graviton background [20].

We can now generalize Eq. (30) to more general baths of gravitons. The energy density of such a bath may be written as (See Eq. (50) in Ref. [15], where $16\pi G = 16\pi\ell_P^2 = 1$ units were used.)

$$\rho_g = \frac{1}{32\pi\ell_P^2} \langle \dot{h}_{ij} \dot{h}^{ij} \rangle = \frac{\omega_g^2}{32\pi\ell_P^2} \langle h_{ij} h^{ij} \rangle. \quad (33)$$

Here ω_g is the characteristic graviton angular frequency. We can use Eq. (27) to write

$$h = \frac{4}{3} \sqrt{2\pi} \frac{\lambda_g \sqrt{\rho_g}}{E_P}, \quad (34)$$

where $\lambda_g = 2\pi/\omega_g$ is the characteristic graviton wavelength. This leads to a decoherence time of

$$t_d = \frac{3}{4\sqrt{2\pi}} \frac{E_P}{\lambda_g \sqrt{\rho_g} \Delta\omega}. \quad (35)$$

Thus a bath of lower frequency gravitons will be more effective in causing decoherence for a fixed graviton energy density. We can express the decoherence time as

$$t_d \approx 2 \times 10^8 \text{yr} \left(\frac{1\text{cm}}{\lambda_g}\right) \left(\frac{\rho_{\text{CMB}}}{\rho_g}\right)^{1/2} \left(\frac{1\text{eV}}{\Delta\omega}\right), \quad (36)$$

where ρ_{CMB} is the energy density associated with the cosmic microwave background at $T \approx 2.7\text{K}$. This time is still quite long unless either λ_g , ρ_g , or the energy difference $\Delta\omega$ are large.

Recall that this estimate holds for the case where $\omega_g > \omega_0$, the graviton frequency is larger than the natural resonant frequency associated with non-gravitational binding forces. In the opposite limit, where $\omega_g < \omega_0$, we see from Eq. (23) that the magnitude of the position fluctuations will be suppressed by a factor of $(\omega_g/\omega_0)^2$. This leads to a corresponding decrease in the decoherence rate, and an increase in the decoherence time by a factor of $(\omega_0/\omega_g)^2$.

IV. SUMMARY

In this paper, we have treated an example of gravitational decoherence, the loss of quantum coherence as a result of gravitational effects. The specific effect we discuss is dephasing due to quantum length fluctuations. This effect was illustrated by an analog model of quantum particles in a box with walls whose positions fluctuate. Quantum geometry fluctuations produced by a bath of gravitons were discussed and shown to have the same effect. The decoherence rate is typically rather small in the present day universe. However, the rate increases with increasing graviton wavelength for fixed energy density, and also increases as the energy differences in the quantum system increase. In any case, this is a simple model which shows that gravitational decoherence is in principle possible, and illustrates the role of spacetime geometry fluctuations.

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